

## DSP: Lec 9

### Realization structures of Digital Filters

"Digital filter structure"

#### \* structure of IIR Filters

Digital Filters  $\Rightarrow$  discrete time (LTI) systems.

$\rightarrow$  these filters are described by difference equations

$\rightarrow$  These systems can be implemented in hardware or software.

$\rightarrow$  There exist various methods to implement these filters, these methods are called Realization for filter structures.

IIR filters / FIR filters

IIR systems / FIR systems.

IIR  $\rightarrow$  Infinite Impulse Response.

FIR  $\rightarrow$  finite

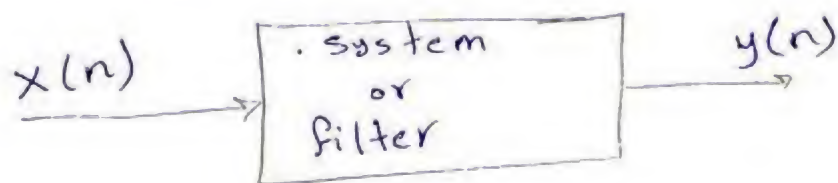
\*

\* IIR systems / filters

$\rightarrow$  general form for the difference eqn:

$$y(n) + a_1 y(n-1) + a_2 y(n-2) + \dots + a_N y(n-N)$$

$$= b_0 x(n) + b_1 x(n-1) + \dots + b_M x(n-M)$$



\* FIR systems / filters

\* general form for the difference equation

$$y(n) = b_0 x(n) + b_1 x(n-1) + \dots + b_M x(n-M)$$

For IIR

~~← IIR~~

$$\underline{\underline{Z.T}} \Rightarrow \left[ 1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N} \right] Y(z)$$

$$= \left[ b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M} \right] X(z)$$



$$T.F = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \dots + b_m z^{-m}}{1 + a_1 z^{-1} + \dots + a_N z^{-N}}$$

a - Feedback system.

b - Consists of Poles & Zeros.

c - depend on the Present and Past i/p's and also Past o/p's.

For FIR systems / Filters

Z.T

$$T.F = \frac{Y(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_n z^{-n}$$

a - The system isn't feedback system.

b - Consists of zeros only.

c - depend on the Present and Past i/p's.

FIR (IIR في a, b, c) النقطة ←

هي الفرق بين ~~الفرق~~ الاثنين.

once upon  
on an exam

\* Recursive Filters  $\equiv$  IIR Filters.

\* Non Recursive Filters  $\equiv$  FIR Filters.

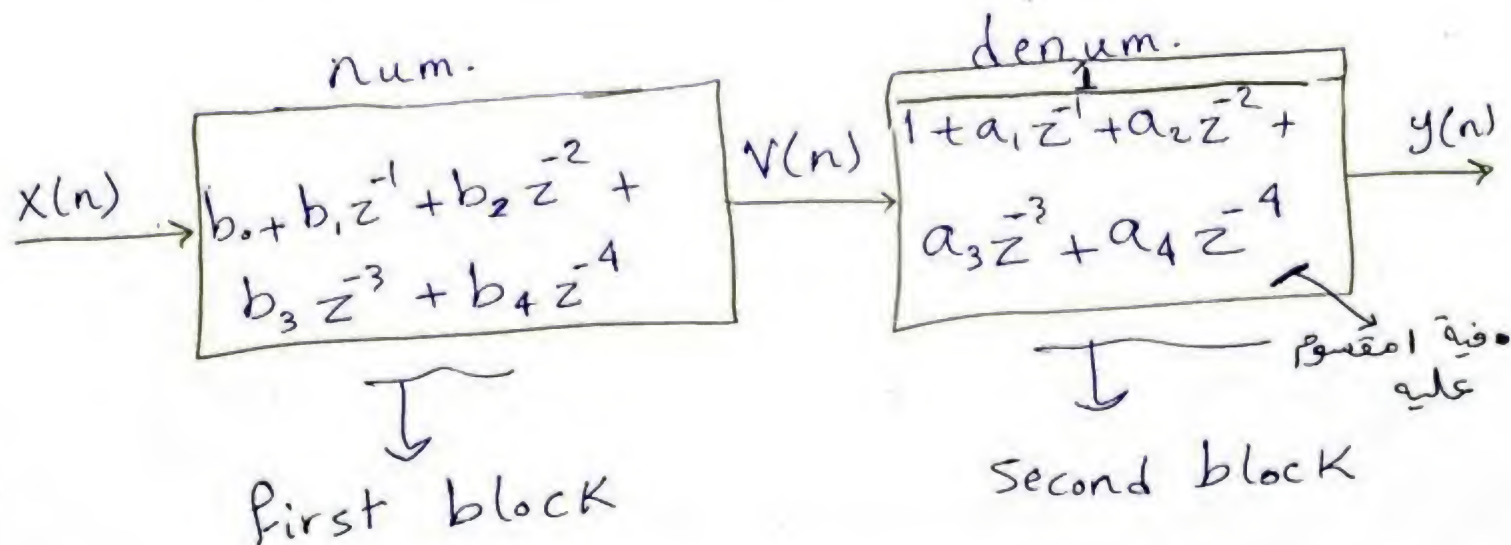
## Digital Filter structures (For IIR Filters)

### II Direct Form I

$$T.F = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}}$$

For  $N = M = 4$

$$T.F = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$



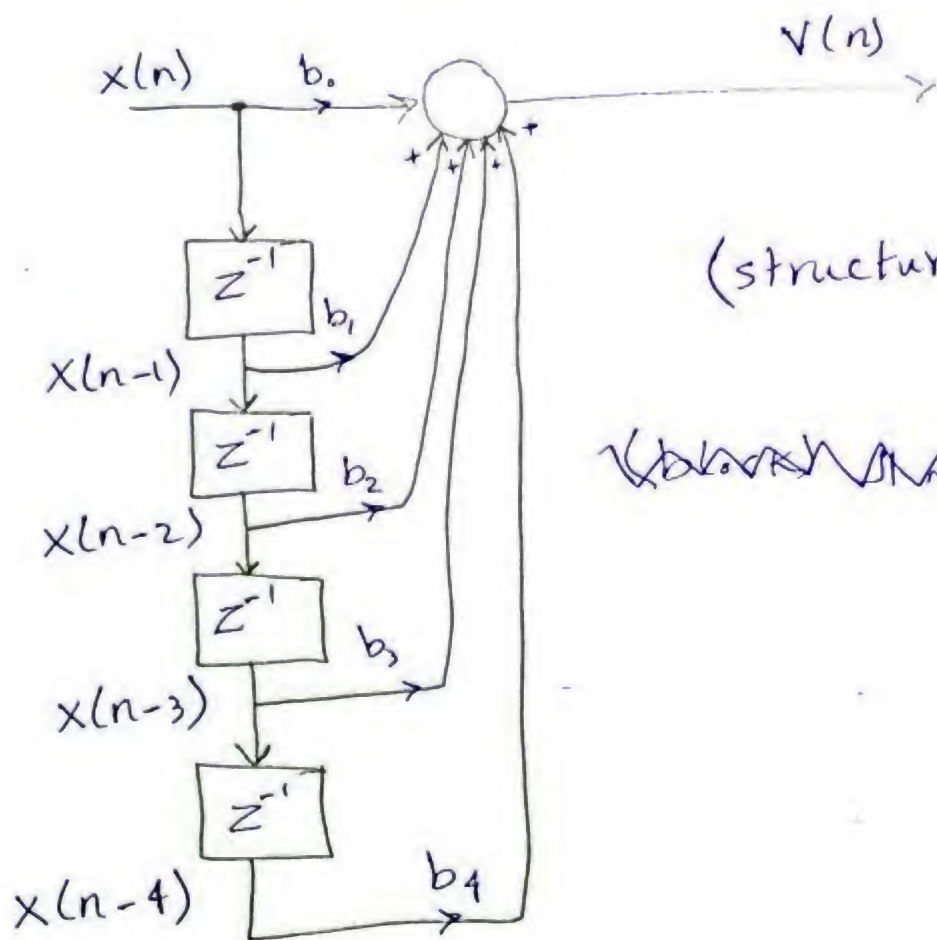


For first block

$$\frac{V(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}$$

$$V(z) = X(z) [b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}] \quad \Downarrow \quad z^{-1} \cdot T$$

$$v(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + b_3 x(n-3) + b_4 x(n-4)$$



به جزء من ال (structure)

يعبر عن البنية .

~~من كل جزء جزء~~

منه

\* For second block

$$\frac{Y(z)}{V(z)} = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$

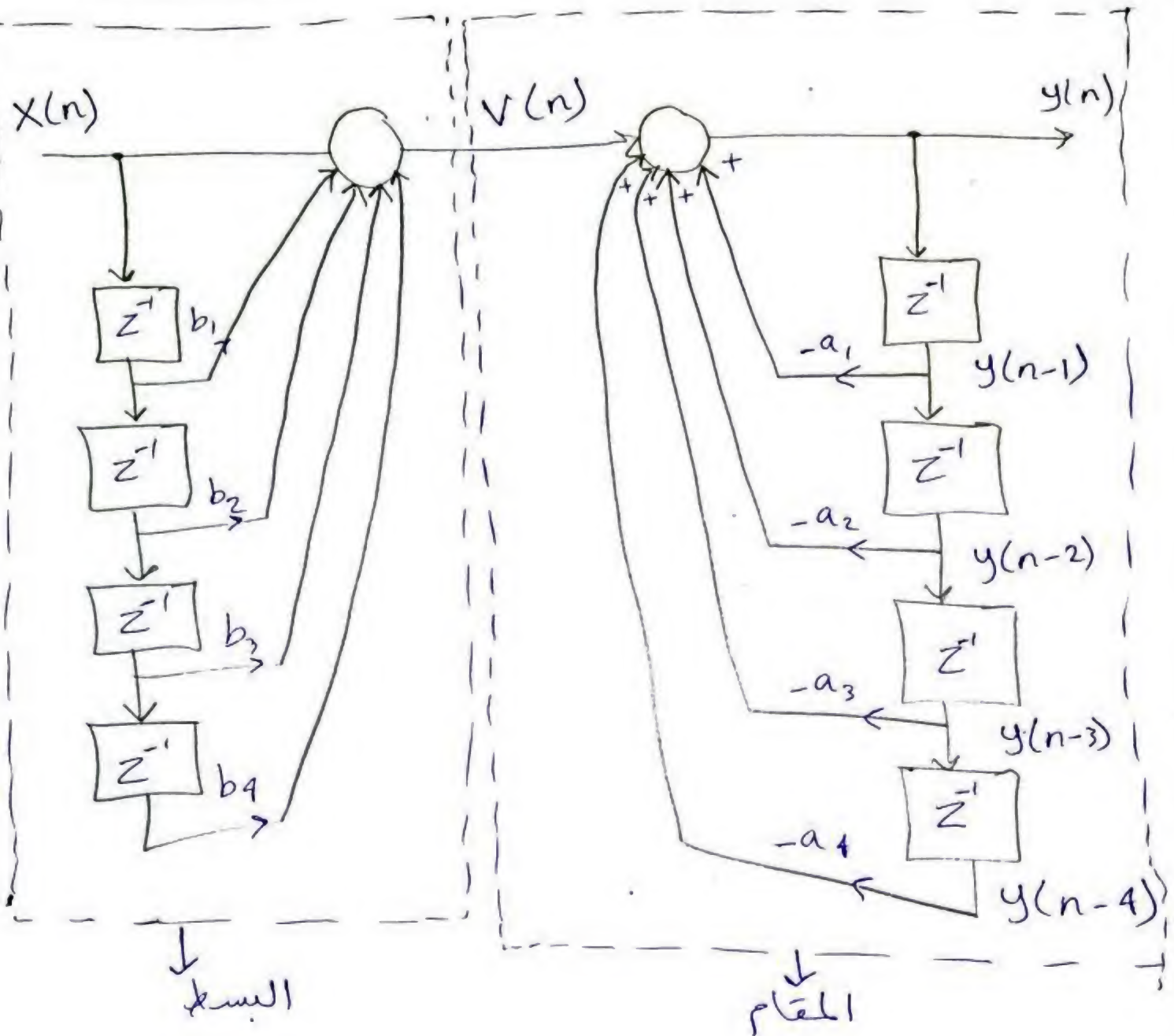
$$V(z) = [1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}] Y(z)$$

$$Y(z) = V(z) - a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z) - a_3 z^{-3} Y(z) - a_4 z^{-4} Y(z)$$

$$\xrightarrow{z^{-1} \cdot T}$$

$$y(n) = v(n) - a_1 y(n-1) - a_2 y(n-2) - a_3 y(n-3) - a_4 y(n-4)$$

← الرسم كامله البسيط، المقام معاً في الصفحة القادمة.

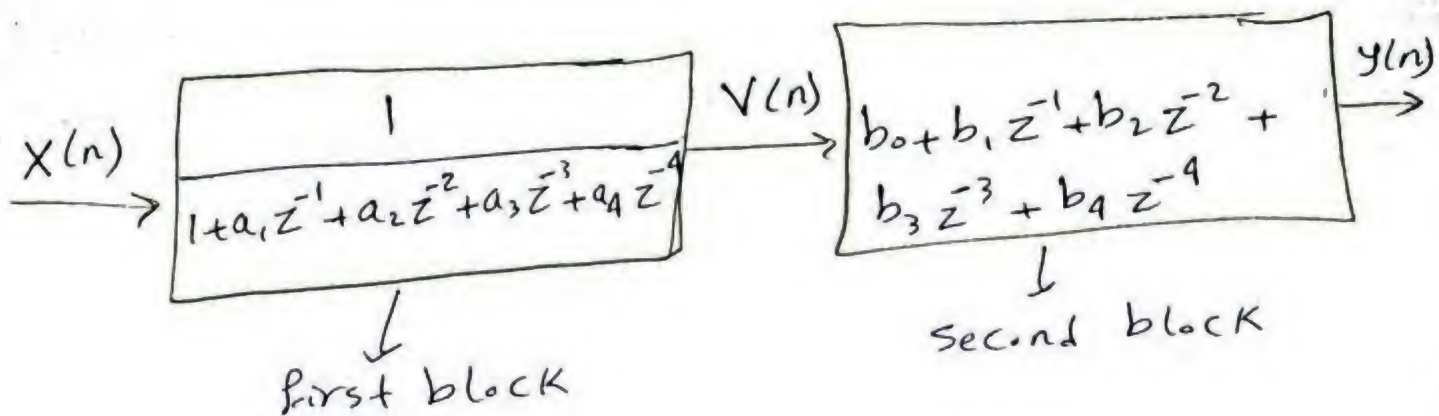


## 2 Direct Form II

for ex:  $N = M = 4$

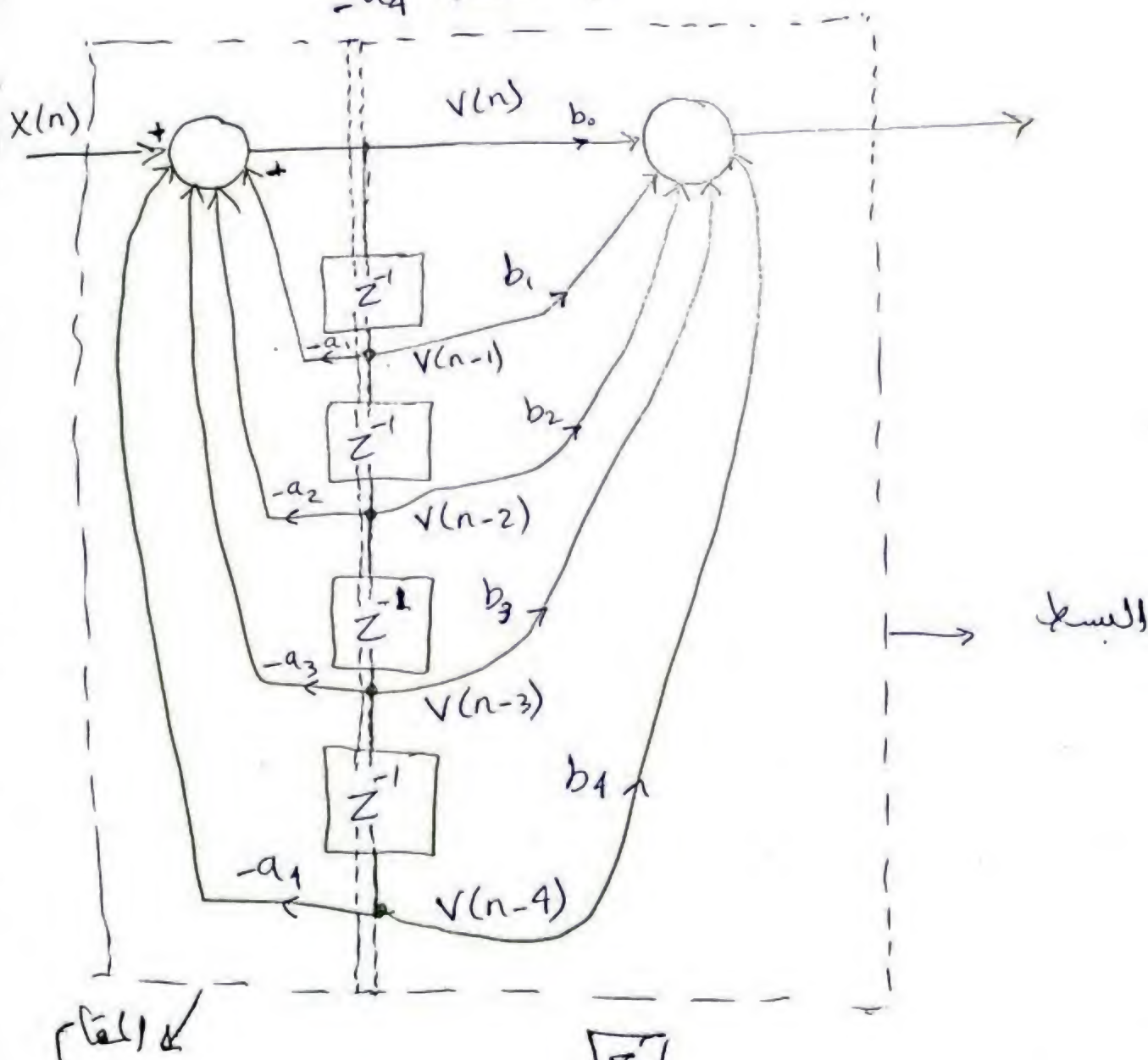
$$T.F = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$





First block

$$v(n) = x(n) - a_1 v(n-1) - a_2 v(n-2) - a_3 v(n-3) - a_4 v(n-4)$$





→ second block

$$T.F = \frac{Y(z)}{V(z)} = b_0 + b_1 z^{-1} + \dots + b_4 z^{-4}$$

$$y(n) = b_0 v(n) + b_1 v(n-1) + b_2 v(n-2) + b_3 v(n-3) + b_4 v(n-4)$$

← الرسمة كاملة في الصفحة السابقة .

Ex For the following Digital Filter

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{(1 + z^{-1})(1 + 0.4 z^{-1})(1 + 0.9 z^{-1})}$$

Draw the corresponding Direct form I & II

$$H(z) = \frac{1 + z^{-2}}{1 + 2.3 z^{-1} + 1.66 z^{-2} + 0.36 z^{-3}}$$

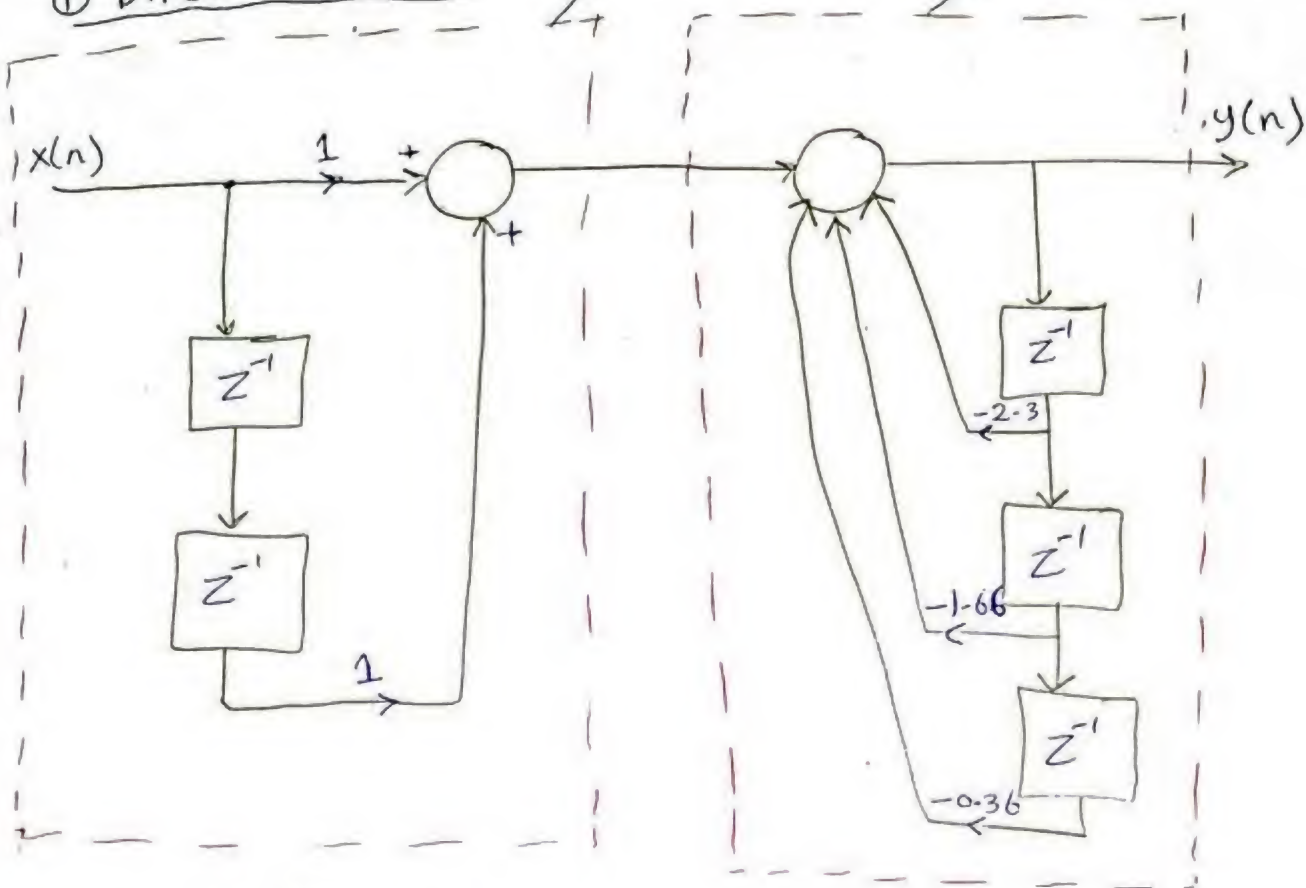
لازم تكون  
ب. 1.

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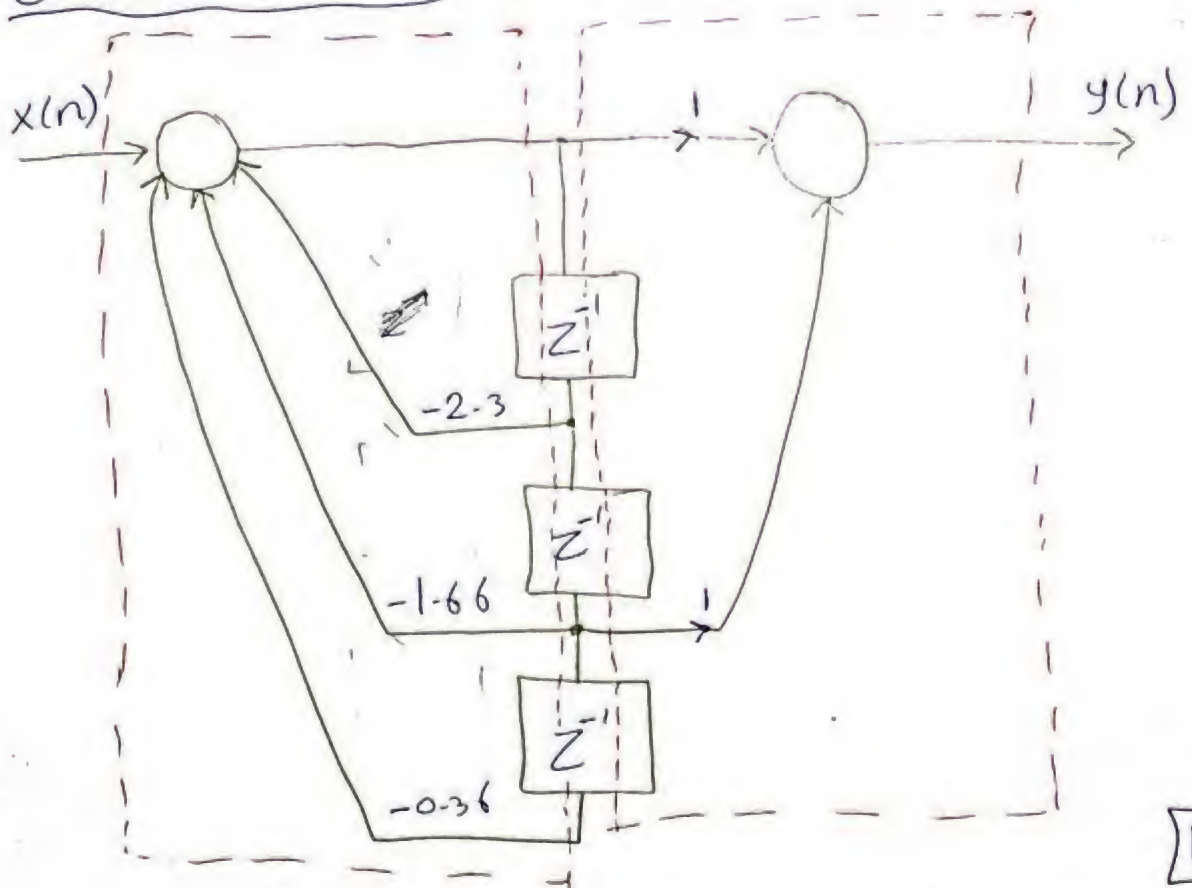
### ① Direct Form I

البسط

المقام



### ② Direct Form II



EX2

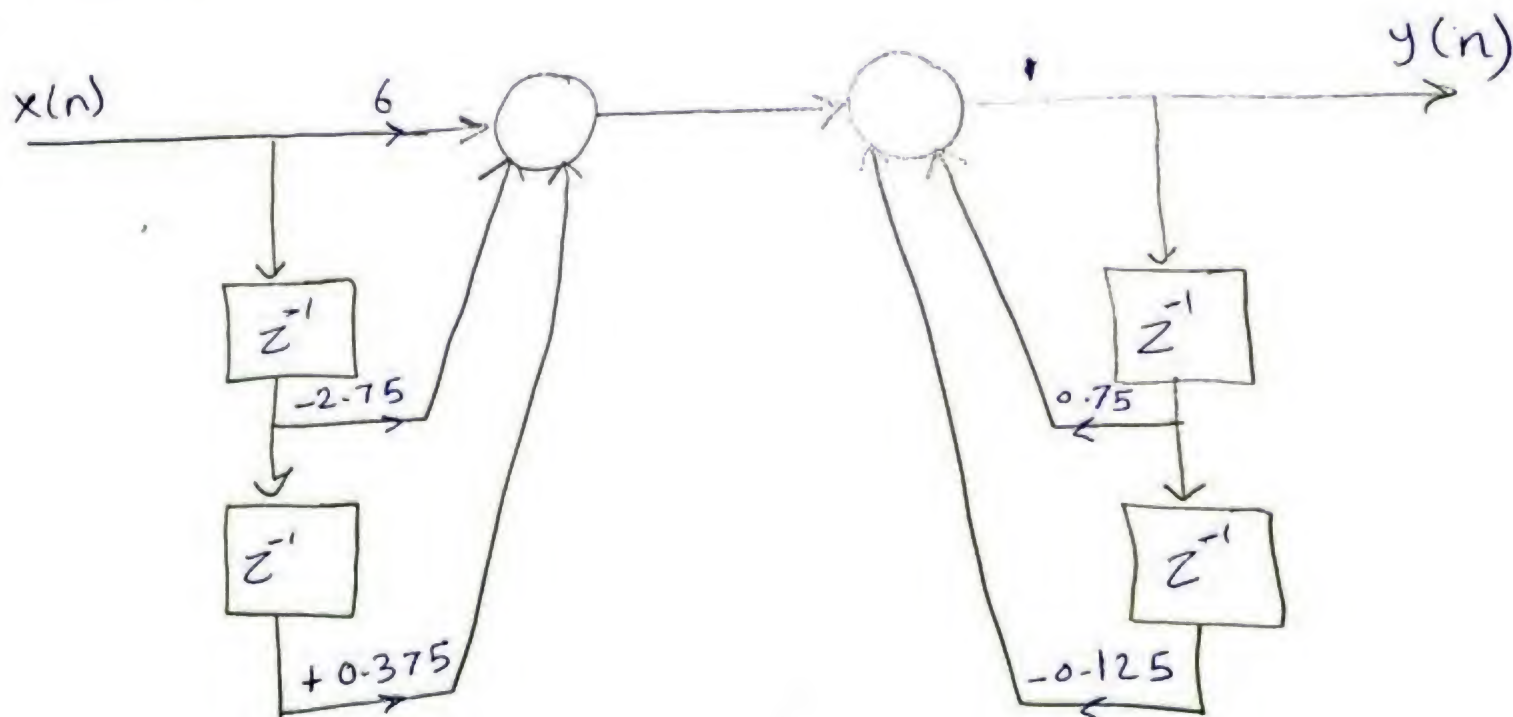
$$H(z) = 3 + \frac{4z}{z-0.5} + \frac{z}{z-0.25}$$

$$= \frac{6z^2 - 2.75z + 0.375}{z^2 - 0.75z + 0.125}$$

$$\times \frac{z^{-2}}{z^{-2}}$$

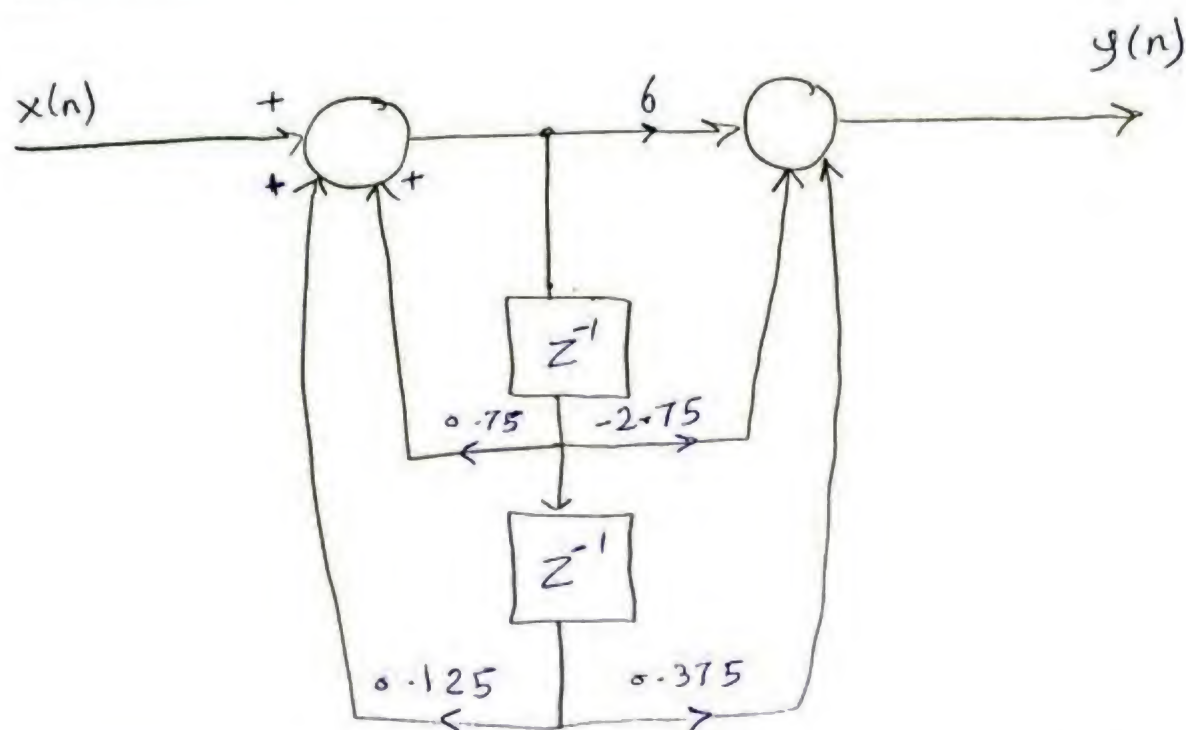
$$= \frac{6 - 2.75z^{-1} + 0.375z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

\* Direct Form I





## \* Direct Form II



## [3] Parallel Form

$$T.F = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$

$$= \frac{\quad}{( \quad ) ( \quad ) ( \quad ) \dots ( \quad )} \quad \Downarrow \text{P.F.}$$

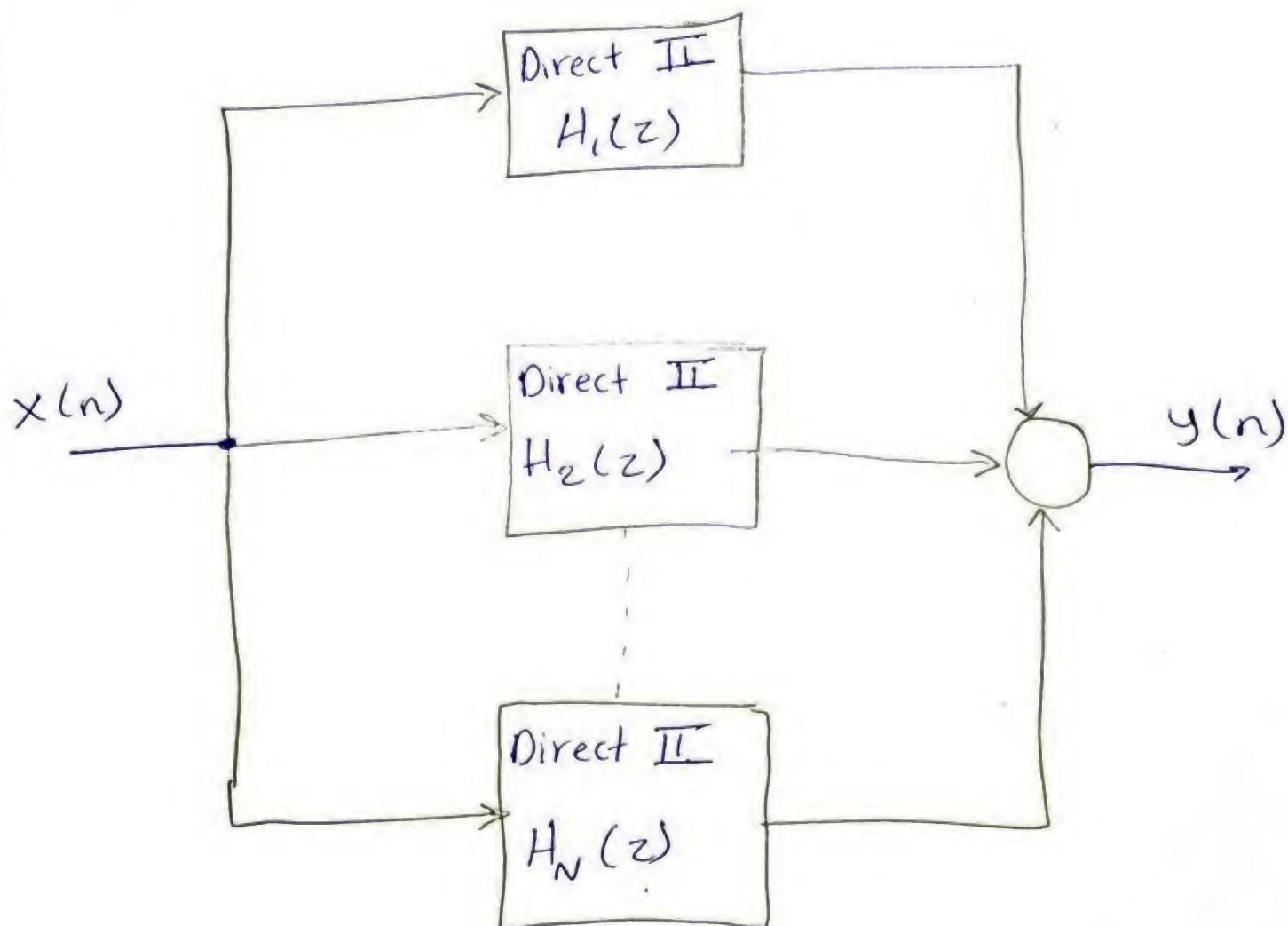
$\uparrow$   
First or second order

$$= \frac{A_1}{( \quad )} + \frac{A_2}{( \quad )} + \dots$$

$$T.F = H_1(z) + H_2(z) + \dots + H_N(z)$$

↓  
Direct Form I or II

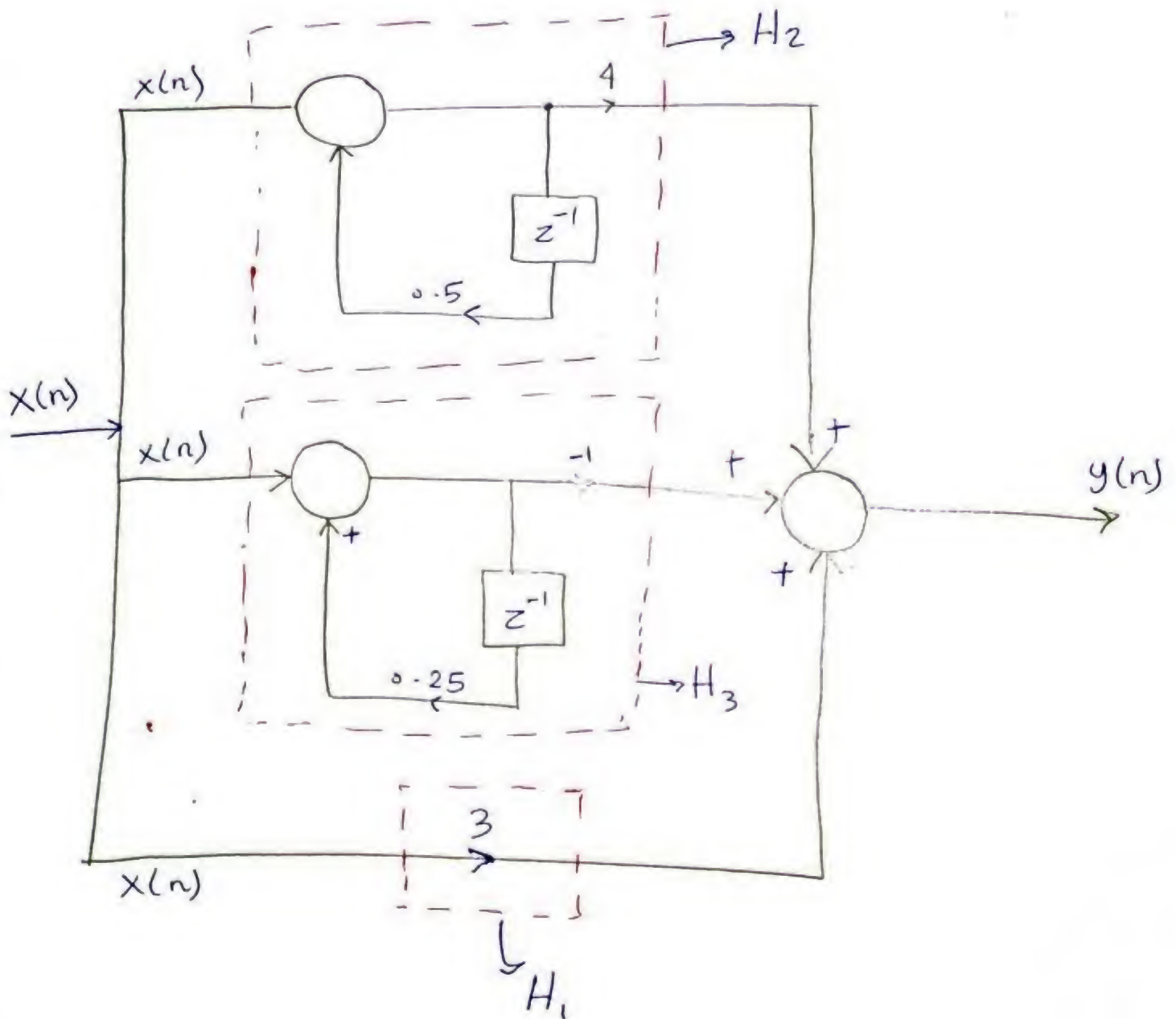
هنا نحلل المقام ثم نقسم للمقام بواسطة (Partial Fraction)  
ثم نفس كل جزء ب  $H(z)$  مثلاً ونحسب (Direct Form)  
(II or I) ونجمع معاً فتدعى الـ (Parallel Form)



**Ex**  $H(z) = 3 + \frac{4z}{z-0.5} - \frac{z}{z-0.25}$

Realize the system using Parallel Form:-

$$H(z) = \underbrace{3}_{H_1} + \underbrace{\frac{4}{1-0.5z^{-1}}}_{H_2} - \underbrace{\frac{1}{1-0.25z^{-1}}}_{H_3}$$





**Ex**

$$H(z) = \frac{1 + z^{-2}}{(1 + z^{-1})(1 + 0.4z^{-1})(1 + 0.9z^{-1})}$$

Draw the structure form in parallel form.

$$\boxed{\text{Put } x = z^{-1}}$$

$$H(z) = \frac{1 + x^2}{(1+x)(1+0.4x)(1+0.9x)} = \frac{A_1}{1+x} + \frac{A_2}{1+0.4x} + \frac{A_3}{1+0.9x}$$

$$A_1 = 33.33, \quad A_2 = 3.87, \quad A_3 = -36.2$$

$$\text{Put } x \longrightarrow z^{-1}$$

■

$$H(z) = \underbrace{\frac{33.33}{1 + z^{-1}}}_{H_1} + \underbrace{\frac{3.87}{1 + 0.4z^{-1}}}_{H_2} - \underbrace{\frac{36.2}{1 + 0.9z^{-1}}}_{H_3}$$

